

AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

- 1 1. (Currently amended) A method for using a computer system to solve an
2 unconstrained interval global optimization problem specified by a function f ,
3 wherein f is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots x_n)$, the method
4 comprising:
5 receiving a representation of the function f at the computer system;
6 storing the representation in a memory within the computer system; and
7 performing an interval global optimization process to compute guaranteed
8 bounds on a globally minimum value of the function $f(\mathbf{x})$ over a subbox \mathbf{X} ;
9 wherein performing the interval global optimization process involves,
10 applying term consistency to a set of relations associated
11 with the function f over the subbox \mathbf{X} , and excluding any portion of
12 the subbox \mathbf{X} that violates any of these relations,
13 applying box consistency to the set of relations associated
14 with the function f over the subbox \mathbf{X} , and excluding any portion of
15 the subbox \mathbf{X} that violates any of these relations, and
16 performing an interval Newton step on the subbox \mathbf{X} to
17 produce a resulting subbox \mathbf{Y} , wherein the point of expansion of
18 the interval Newton step is a point \mathbf{x} within the subbox \mathbf{X} , and
19 wherein performing the interval Newton step involves evaluating
20 | the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$ using interval arithmetic; and

21 | recording the guaranteed bounds in the memory within the computer
22 | system.

1 2. (Original) The method of claim 1, wherein applying term consistency
2 involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a term $g(x_j)$, thereby producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein
5 the term $g(x_j)$ can be analytically inverted to produce an inverse function $g^{-1}(y)$;
6 substituting the subbox \mathbf{X} into the modified equation to produce the
7 equation $g(X'_j) = h(\mathbf{X})$;
8 solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and
9 intersecting X'_j with the interval X_j to produce a new subbox \mathbf{X}^+ ;
10 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
11 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
12 the size of the subbox \mathbf{X} .

1 3. (Original) The method of claim 1, wherein performing the interval
2 global optimization process involves:
3 keeping track of a smallest upper bound f_bar of the function $f(\mathbf{x})$;
4 removing from consideration any subbox \mathbf{X} for which $f(\mathbf{X}) > f_bar$; and
5 wherein applying term consistency to the f_bar relation involves applying
6 term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 4. (Original) The method of claim 3, wherein applying box consistency to
2 the set of relations involves applying box consistency to the inequality $f(\mathbf{x}) \leq$
3 f_bar over the subbox \mathbf{X} .

1 5. (Original) The method of claim 1, wherein performing the interval
2 global optimization process involves:
3 determining the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
4 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
5 removing from consideration any subbox for which any element of $\mathbf{g}(\mathbf{x})$ is
6 bounded away from zero, thereby indicating that the subbox does not include a
7 stationary point of $f(\mathbf{x})$; and
8 wherein applying term consistency to the set of relations involves applying
9 term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox
10 \mathbf{X} .

1 6. (Original) The method of claim 5, wherein applying box consistency to
2 the set of relations involves applying box consistency to each component
3 $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox \mathbf{X} .

1 7. (Original) The method of claim 1, wherein performing the interval
2 global optimization process involves:
3 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
4 function $f(\mathbf{x})$;
5 removing from consideration any subbox for which a diagonal element of
6 the Hessian is always negative, which indicates that the function f is not convex
7 and consequently does not contain a global minimum within the subbox;
8 wherein applying term consistency to the set of relations involves applying
9 term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 8. (Original) The method of claim 7, wherein applying box consistency to
2 the set of relations involves applying box consistency to each inequality
3 $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 9. (Original) The method of claim 1,
2 wherein performing the interval Newton step involves,
3 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient \mathbf{g} evaluated
4 as a function of a point \mathbf{x} over the subbox \mathbf{X} ,
5 computing an approximate inverse \mathbf{B} of the center of
6 $\mathbf{J}(\mathbf{x}, \mathbf{X})$, and
7 using the approximate inverse \mathbf{B} to analytically determine
8 the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$,
9 and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
10 wherein applying term consistency to the set of relations involves applying
11 term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable
12 x_i ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 10. (Original) The method of claim 9, wherein applying box consistency to
2 the set of relations involves applying box consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i$
3 $= 0$ ($i=1, \dots, n$) for each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 11. (Original) The method of claim 1, further comprising terminating
2 attempts to further reduce the subbox \mathbf{X} when:
3 the width of \mathbf{X} is less than a first threshold value; and
4 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

1 12. (Original) The method of claim 11, wherein performing the interval
2 Newton step involves:
3 computing $\mathbf{J}(\mathbf{x}, \mathbf{X})$, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the function \mathbf{f}
4 evaluated as a function of \mathbf{x} over the subbox \mathbf{X} ; and
5 determining if $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular as a byproduct of solving for the subbox \mathbf{Y}
6 that contains values of \mathbf{y} that satisfy $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y}-\mathbf{x}) = \mathbf{r}(\mathbf{x})$, where

7 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
8 $\mathbf{J}(\mathbf{x}, \mathbf{X})$.

1 13. (Original) A computer-readable storage medium storing instructions
2 that when executed by a computer cause the computer to perform a method for
3 using a computer system to solve an unconstrained interval global optimization
4 problem specified by a function f , wherein f is a scalar function of a vector
5 $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the method comprising:
6 receiving a representation of the function f at the computer system;
7 storing the representation in a memory within the computer system; and
8 performing an interval global optimization process to compute guaranteed
9 bounds on a globally minimum value of the function $f(\mathbf{x})$ over a subbox \mathbf{X} ;
10 wherein performing the interval global optimization process involves,
11 applying term consistency to a set of relations associated
12 with the function f over the subbox \mathbf{X} , and excluding any portion of
13 the subbox \mathbf{X} that violates any of these relations,
14 applying box consistency to the set of relations associated
15 with the function f over the subbox \mathbf{X} , and excluding any portion of
16 the subbox \mathbf{X} that violates any of these relations, and
17 performing an interval Newton step on the subbox \mathbf{X} to
18 produce a resulting subbox \mathbf{Y} , wherein the point of expansion of
19 the interval Newton step is a point \mathbf{x} within the subbox \mathbf{X} , and
20 wherein performing the interval Newton step involves evaluating
21 the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$ using interval arithmetic.

1 14. (Original) The computer-readable storage medium of claim 13,
2 wherein applying term consistency involves:

3 symbolically manipulating an equation within the computer system to
4 solve for a term $g(x_j)$, thereby producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein
5 the term $g(x_j)$ can be analytically inverted to produce an inverse function $g^{-1}(y)$;
6 substituting the subbox \mathbf{X} into the modified equation to produce the
7 equation $g(X'_j) = h(\mathbf{X})$;
8 solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and
9 intersecting X'_j with the interval X_j to produce a new subbox \mathbf{X}^+ ;
10 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
11 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
12 the size of the subbox \mathbf{X} .

1 15. (Original) The computer-readable storage medium of claim 13,
2 wherein performing the interval global optimization process involves:
3 keeping track of a smallest upper bound f_bar of the function $f(\mathbf{x})$;
4 removing from consideration any subbox \mathbf{X} for which $f(\mathbf{X}) > f_bar$; and
5 wherein applying term consistency to the f_bar relation involves applying
6 term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 16. (Original) The computer-readable storage medium of claim 15,
2 wherein applying box consistency to the set of relations involves applying box
3 consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 17. (Original) The computer-readable storage medium of claim 13,
2 wherein performing the interval global optimization process involves:
3 determining the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
4 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);

5 removing from consideration any subbox for which any element of $\mathbf{g}(\mathbf{x})$ is
6 bounded away from zero, thereby indicating that the subbox does not include a
7 stationary point of $f(\mathbf{x})$; and
8 wherein applying term consistency to the set of relations involves applying
9 term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox
10 \mathbf{X} .

1 18. (Original) The computer-readable storage medium of claim 17,
2 wherein applying box consistency to the set of relations involves applying box
3 consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox \mathbf{X} .

1 19. (Original) The computer-readable storage medium of claim 13,
2 wherein performing the interval global optimization process involves:
3 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
4 function $f(\mathbf{x})$;
5 removing from consideration any subbox for which a diagonal element of
6 the Hessian is always negative, which indicates that the function f is not convex
7 and consequently does not contain a global minimum within the subbox;
8 wherein applying term consistency to the set of relations involves applying
9 term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 20. (Original) The computer-readable storage medium of claim 19,
2 wherein applying box consistency to the set of relations involves applying box
3 consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 21. (Original) The computer-readable storage medium of claim 13,
2 wherein performing the interval Newton step involves,

3 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient \mathbf{g} evaluated
4 as a function of a point \mathbf{x} over the subbox \mathbf{X} ,
5 computing an approximate inverse \mathbf{B} of the center of
6 $\mathbf{J}(\mathbf{x}, \mathbf{X})$, and
7 using the approximate inverse \mathbf{B} to analytically determine
8 the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$,
9 and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
10 wherein applying term consistency to the set of relations involves applying
11 term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable
12 x_i ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 22. (Original) The computer-readable storage medium of claim 21,
2 wherein applying box consistency to the set of relations involves applying box
3 consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable x_i
4 ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 23. (Original) The computer-readable storage medium of claim 13,
2 wherein the method further comprises terminating attempts to further reduce the
3 subbox \mathbf{X} when:
4 the width of \mathbf{X} is less than a first threshold value; and
5 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

1 24. (Original) The computer-readable storage medium of claim 13,
2 wherein performing the interval Newton step involves:
3 computing $\mathbf{J}(\mathbf{x}, \mathbf{X})$, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the function \mathbf{f}
4 evaluated as a function of \mathbf{x} over the subbox \mathbf{X} ; and
5 determining if $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular as a byproduct of solving for the subbox \mathbf{Y}
6 that contains values of \mathbf{y} that satisfy $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$, where

7 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
8 $\mathbf{J}(\mathbf{x}, \mathbf{X})$.

1 25. (Currently amended) An apparatus that solves an unconstrained
2 interval global optimization problem specified by a function f , wherein f is a scalar
3 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the apparatus comprising:
4 a receiving mechanism that is configured to receive a representation of the
5 function f ;
6 a memory for storing the representation; and
7 an interval global optimization mechanism that is configured to perform
8 an interval global optimization process to compute guaranteed bounds on a
9 globally minimum value of the function $f(\mathbf{x})$ over a subbox \mathbf{X} ;
10 a term consistency mechanism within the interval global optimization
11 mechanism that is configured to apply term consistency to a set of relations
12 associated with the function f over the subbox \mathbf{X} , and to exclude any portion of the
13 subbox \mathbf{X} that violates any of these relations;
14 a box consistency mechanism within the interval global optimization
15 mechanism that is configured to apply box consistency to the set of relations
16 associated with the function f over the subbox \mathbf{X} , and to exclude any portion of the
17 subbox \mathbf{X} that violates any of these relations; and
18 an interval Newton mechanism within the interval global optimization
19 mechanism that is configured to perform an interval Newton step on the subbox \mathbf{X}
20 to produce a resulting subbox \mathbf{Y} , wherein the point of expansion of the interval
21 Newton step is a point \mathbf{x} within the subbox \mathbf{X} , and wherein performing the interval
22 Newton step involves evaluating the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$ using
23 interval arithmetic; and
24 a recording mechanism configured to record the guaranteed bounds in the
25 memory.

1 26. (Original) The apparatus of claim 25, wherein the term consistency
2 mechanism is configured to:
3 symbolically manipulate an equation to solve for a term $g(x_j)$, thereby
4 producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein the term $g(x_j)$ can be
5 analytically inverted to produce an inverse function $g^{-1}(y)$;
6 substitute the subbox \mathbf{X} into the modified equation to produce the equation
7 $g(X'_j) = h(\mathbf{X})$;
8 solve for $X'_j = g^{-1}(h(\mathbf{X}))$; and to
9 intersect X'_j with the interval X_j to produce a new subbox \mathbf{X}^+ ;
10 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
11 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
12 the size of the subbox \mathbf{X} .

1 27. (Original) The apparatus of claim 25,
2 wherein the interval global optimization mechanism is configured to,
3 keep track of a smallest upper bound f_bar of the function
4 $f(\mathbf{x})$, and to
5 remove from consideration any subbox \mathbf{X} for which
6 $f(\mathbf{X}) > f_bar$; and
7 wherein the term consistency mechanism is configured to apply term
8 consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 28. (Original) The apparatus of claim 27, wherein the box consistency
2 mechanism is configured to apply box consistency to the inequality $f(\mathbf{x}) \leq f_bar$
3 over the subbox \mathbf{X} .

1 29. (Original) The apparatus of claim 25,
2 wherein the interval global optimization mechanism is configured to,

3 determine the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein
4 $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$), and to
5 remove from consideration any subbox for which any
6 element of $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that
7 the subbox does not include a stationary point of $f(\mathbf{x})$; and
8 wherein the term consistency mechanism is configured to apply term
9 consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox \mathbf{X} .

1 30. (Original) The apparatus of claim 29, wherein the box consistency
2 mechanism is configured to apply box consistency to each component
3 $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox \mathbf{X} .

1 31. (Original) The apparatus of claim 25,
2 wherein the interval global optimization mechanism is configured to,
3 determine diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the
4 Hessian of the function $f(\mathbf{x})$, and to
5 remove from consideration any subbox for which a
6 diagonal element of the Hessian is always negative, which
7 indicates that the function f is not convex and consequently does
8 not contain a global minimum within the subbox;
9 wherein the term consistency mechanism is configured to apply term
10 consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 32. (Original) The apparatus of claim 31, wherein the box consistency
2 mechanism is configured to apply box consistency to each inequality
3 $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 33. (Original) The apparatus of claim 25,

2 wherein the interval Newton mechanism is configured to,
3 compute the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient \mathbf{g} evaluated as
4 a function of a point \mathbf{x} over the subbox \mathbf{X} ,
5 compute an approximate inverse \mathbf{B} of the center of $\mathbf{J}(\mathbf{x}, \mathbf{X})$,
6 and to
7 use the approximate inverse \mathbf{B} to analytically determine the
8 system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and
9 wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
10 wherein the term consistency mechanism is configured to apply term
11 consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable
12 x_i ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 34. (Original) The apparatus of claim 33, wherein the box consistency
2 mechanism is configured to apply box consistency to each component
3 $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 35. (Original) The apparatus of claim 25, further comprising a termination
2 mechanism that is configured to terminate attempts to further reduce the subbox \mathbf{X}
3 when:
4 the width of \mathbf{X} is less than a first threshold value; and
5 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

1 | 36. (Currently amended) The apparatus of ~~claim 11~~ claim 35, wherein the
2 interval Newton mechanism is configured to:
3 compute $\mathbf{J}(\mathbf{x}, \mathbf{X})$, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the function \mathbf{f} evaluated
4 as a function of \mathbf{x} over the subbox \mathbf{X} ; and to
5 determine if $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular as a byproduct of solving for the subbox \mathbf{Y}
6 that contains values of \mathbf{y} that satisfy $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$, where

- 7 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
- 8 $\mathbf{J}(\mathbf{x}, \mathbf{X})$.